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Algorithmic Aspects of Vertex Geo-dominating Sets and Geo-number in Graphs

Dr. A. Anto Kinsley* K. Karthika**
* (Associate Professor, Department of Mathematics, St. Xavier's College (Autonomous), Palayamkottai - 627 002, India)
** (Research Scholar, St. Xavier's College (Autonomous), Palayamkottai - 627 002, India)

ABSTRACT
In this paper we study about *x*-geodominating set, geodetic set, geo-set, geo-number of a graph G. We study the binary operation, link vectors and some required results to develop algorithms. First we design two algorithms to check whether given set is an *x*-geodominating set and to find the minimum *x*-geodominating set of a graph. Finally we present another two algorithms to check whether a given vertex is geo-vertex or not and to find the geo-number of a graph.
Key words: Graph, *x*-geodominating set, geodetic set, geo-set, geo- number, link vector, and graph algorithms.

I. INTRODUCTION

By a graph $G = (V, E)$, we mean a finite, undirected, connected graph without loop or multiple edges [1]. For a graph theoretic terminology we refer to the book by F. Harary and Buckley [2]. The geodetic number of a graph was introduced in [2] and further studied in [3]. Geodetic concepts were first studied from the point of view of domination by Chartrand, Harary, Swart and Zhang in [4] and further studied by several others. The concept of vertex geodomination number and geo-number was introduced by Santhakumaran and Titus [5], [6], [7]. We assume that $|V| = n$ throughout this paper. Before we present the algorithm, we give a brief description of the computation of link vector [8] of the closed interval of the graphs that are involved in our algorithms.

In this paper, we design four algorithms. First we present two algorithms (i) to check whether a given set of vertices is an *x*-geodominating set and (ii) to find the minimum *x*-geodominating set of a graph. Finally we present we two algorithms to check whether a given vertex is geo-vertex or not and an algorithm to find the geo-number of a graph. In all algorithms we consider a graph with distance matrix.

In this section, some definition and important results on *x*-geodominating sets and geo-set are given.

Definition 1.1

For a connected graph G of order $p \geq 2$, a set $S \subseteq V(G)$ is an *x-geodominating set* of G if each vertex $v \in V(G)$ lies on an *x-y* geodesic for some element y in S. The minimum cardinality of an *x*-geodominating set of G is defined as the *x-geodomination number* of G and denoted by $g_x(G)$. The *x*-geodominating set of cardinality $g_x(G)$ is called a g_x*- set* or *minimum x-geodominating set* of G.

Definition 1.2

For a connected graph G of order $p \leq 2$, a vertex x is called a *geo-vertex* if S_x is a g_x*-set* and $S_x \cup \{x\}$ is a g-set of G. The set of all geo-vertices is called as *geo-set* and the cardinality of the geo-set is called as *geo-number* $gn(G)$.

Example 1.3

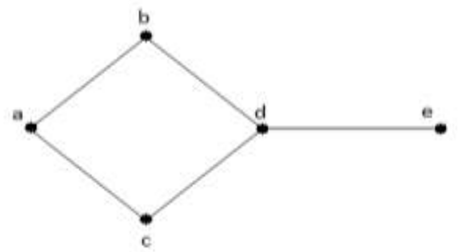

Figure 1: A graph *G*

Vertex x	Vertex Geodominating Sets S_x	$g_x\ (G)$
a	$\{e\}$	1
b	$\{e, c\}$	2
c	$\{e, b\}$	2
d	$\{a, e\}$	2
e	$\{a\}$	1

Table 1

The table 1 gives the vertex geodominating sets of a graph G given in figure 1. The set $S_a \cup \{a\}$ and $S_e \cup \{e\}$ forms the g-set of G. So the vertices a and d are called geo-vertex and the geo-number is 2 for the graph G.

II. LINK VECTORS

Definition 2.1

Characterize each closed interval as an n-tuple. Each place of n-tuple can be represented by a binary 1 or 0. Call this n-tuple as a *link vector*. Denote $LV\ (I) = I'$. Put 1 if the vertex belongs to the closed interval otherwise 0. If all the co-ordinate of the link vector is equal to 1 then it is called as *full*. Denote $I\ [(1)]$.

Definition 2.2

Let G be a graph. Let ρ be the set of all LV of G. Define a binary operation $\vee: \rho \times \rho \to \rho$ by $(v_1, v_2, \ldots, v_k) \vee (u_1, u_2, \ldots, u_k) = (w_1, w_2, \ldots, w_k)$, where $w_i = \max\ \{v_i, u_i\}$. Now we generalize this idea for more than two LVs. Operation on any number of LVs by $\vee$ can be followed by pairwise.
For any $I_i \in \rho\ (1 \le i \le 4)$, $I'_1 \vee I'_2 \vee I'_3$ means $(I'_1 \vee I'_2) \vee I'_3$ or $I'_1 \vee (I'_2 \vee I'_3)$.
$I'_1 \vee I'_2 \vee I'_3 \vee I'_4$ means $(I'_1 \vee I'_2) \vee (I'_3 \vee I'_4)$ and so on.

Theorem 2.3

Let G be a graph with n vertices. Then $\bigvee_{i=1}^{r} I'_i$ is full, where r is the number of closed interval obtained between each pair of vertices of S if and only if $S = \{v_1, v_2, \ldots, v_k\}$ is a geodetic set.

Proof: [9].

III. DEVELOPMENT OF ALGORITHMS

In this section we give an algorithm to find the minimum x-geodominating set and the geo-number of a graph G. The algorithms closed interval $I\ [S_i]$, Link vector $I'\ [S_2]$ and geodetic $[S_j]$ used in these section are studied in [9].

Algorithm 3.1

The x-geodominating set confirmation algorithm:

Input: A graph $G = (V, E)$ and a subset $S = \{v_1, v_2, \ldots, v_k\}$ of vertices and a vertex $v \in V - S$.

Output: S is an x-geodominating set or not.

Take $L \leftarrow (0)$
for $i = 1$ to k
begin
$S_i = \{x, v_i\}$
closed- interval $I_i[S_2]$
link vector $I'_i[S_2]$
$L = L \vee I'_i[S_2]$
end

If L is full then the given set is an x-geodominating set.
Otherwise S is not an x-geodominating set.

In this algorithm, step 2 will work in k times. Next part of this algorithm calls the algorithms closed interval $I\ [S_2]$ and link vector $I'\ [S_2]$ and hence this part will work with $2m+n$ verifications, where m is the number of edges and n is the number of vertices. Thus this algorithm requires $O\ (k(m+n))$ cost of time. But in that step the given vertex acts as a root and all other vertices are approached through a spanning tree. Therefore there are $n + (n-1)$ verifications needed, since $m = n-1$ for a tree. Total cost of time is $O\ (k(n+(n-1))$, that is $O\ (n\ (2n-1))$, that is $O\ (n^2)$. Thus this algorithm requires $O\ (n^2)$ cost of time.

Algorithm 3.2

The minimum x-geodominating set algorithm:

Input: A graph $G = (V, E)$ with $V\ (G) = \{v_1, v_2, \ldots, v_k\}$ of vertices and $x \in V$

Output: S_j's with $g_x(G)$ vertices.

Step 1: Take $k \leftarrow 1$

Step 2: Take all the subsets S_j $\left(1 \le j \le \binom{n}{k}\right)$ of V with k vertices.

Step 3: Check whether any S_j is an x-geodominating set or not using algorithm 3.1.

Step 4: If any such S_j is an x-geodominating set then find all the x-geodominating sets with k vertices using step 3 for all S_j.
4.1: Conclude that the above x-geodominating sets S_j's are minimum.
Stop.
Otherwise continue.

Step 5: Take $k = k + 1$ and return to step 2.

In this algorithm, we will work all subsets of V and hence it is a NP-complete problem.
Next we develop an algorithm to check whether a given vertex is a geo-vertex or not.

Algorithm 3.3

The geo-vertex confirmation algorithm:

Input: A graph $G = (V, E)$ with a vertex v_i

Output: Whether given vertex v_i is geo-vertex or not.
Procedure geo-vertex $[v_i]$;

Step 1: Take $k \leftarrow 1$

Step 2: Take all the subsets S_j $\left(1 \le j \le \binom{n}{k}\right)$ of V with k vertices..

Step 3: Check whether any S_j is a v_i-geodominating set or not using algorithm 3.1.

Step 4: If any such S_j is a v_i-geodominating set then find all the v_i-geodominating sets with k vertices using step 3 for all S_j.

4.1: Check any $S_j \cup \{v_i\}$ obtained from step 4 is a g-set or not using $g$eodetic [S_j].

Step 5: If yes, then v_i is a geo-vertex.
Stop.
Otherwise continue.
Step 6: Take $k = k + 1$ and return to step 2.

In this algorithm, we will work all subsets of V and hence it is a *NP*-complete problem.
Finally we give an algorithm to find the geo-set and geo-number of a graph.

Algorithm 3.4

Algorithm to find a geo-number of a graph:

Input: A graph $G = (V, E)$ with its vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$.

Output: A geo-number $gn(G)$ of a graph.
Procedure geo-number [gn]

Let $S = \varphi$ and count = 0
for i = 1 to n
Geo-vertex [v_i]
if yes then $S = S \cup \{v_i\}$ and count = count +1

The set S is the geo-set and count gives the geo-number of a graph.

In this algorithm, we will work all subsets of V and hence it is a *NP*-complete problem.

IV. CONCLUSION

In this paper we have studied the minimum x-geodominating set and geo-set of a finite, undirected, connected graph without loops or multiple edges, whose distance matrix is known. We have investigated link vectors and the binary operation $\vee$.Some results play an important role in the algorithm development are given. We have initially presented two algorithms (i) to check whether a given set of vertices is an x-geodominating set and (ii) to find the minimum x-geodominating set of a graph. Finally we presented two algorithms to check whether a given vertex is geo-vertex or not and another algorithm to find geo-number of a graph.

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